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Type II Supernova (Cont'd):

Photodisintegration and Neutronization:

At temperatures relevant to our discussion disintegration of Iron nuclei and neutronization can occur and create further instabilities, thus making the collapse more rapid.

Lets start with the photodisintegration. Highly energetic photons can dissociate the Iron nuclei to α particles and neutrons:

$$\gamma + {}^{56}Fe \rightleftarrows 13\alpha + 4n$$

The threshold energy for this process is:

$$\boxed{Q = (13m_\alpha + 4m_n - m_{Fe})c^2 = 124.4 \text{ MeV}}$$

In equilibrium, the number densities of particles participating in this process obey a Saha-like equation:

$$\frac{n_\alpha^{13} n_n^4}{n_{Fe}} = \frac{g_\alpha^{13} g_n^4}{g_{Fe}} \left[\left(\frac{k_B T}{2\pi\hbar^2}\right)^{3/2}\right]^{16} \left(\frac{m_\alpha^{13} m_n^4}{m_{Fe}}\right) \exp\left(-\frac{Q}{k_B T}\right)$$

Here $g_\alpha = 1$, $g_n = 2$, $g_{Fe} = 1.4$ are the corresponding degeneracy factors. In equilibrium, we also have $13 n_n = 4 n_\alpha$. Using this and the above equation, we can find a relation between T and ρ for a given degree of dissociation. In particular, half of the mass of the system will be in the form of ^{56}Fe for:

$$\log \rho = 11.62 + 1.5 \log T_9 - \frac{39.17}{T_9} \qquad \left(T_9 \equiv \frac{T}{10^9 K}\right)$$

For $\rho \simeq 3.7 \times 10^9 \ g\,cm^{-3}$, the characteristic temperature at which the photodisintegration of Iron becomes important is $T \simeq 1.1 \times 10^{10} K$.

At somewhat higher temperatures (and similar densities), the α particles will also dissociate:

$$\gamma + \alpha \rightleftharpoons 2p + 2n$$

The threshold for this process is $\underline{\underline{Q' = 28.3 \ MeV}}$, and the corresponding equation is:

$$\frac{n_p^2 n_n^2}{n_\alpha} = 2\left(\frac{m_u k_B T}{2\pi \hbar^2}\right)^{\frac{9}{2}} \exp\left(-\frac{Q'}{k_B T}\right)$$

For $\rho \simeq 10^9\ g\ cm^{-3}$, the temperature at which ^4He is split into nucleons is $T \simeq 1.5 \times 10^{10}$ K. The fact that this is higher than the temperature for Iron dissociation, despite Q' being smaller than Q, can be understood as follows. The ratio $\frac{n_\alpha}{n_{Fe}}$ has one exponential suppression $\propto \exp(-Q)$, and $\frac{n_N}{n_\alpha}$ (N being a nucleon) has one exponential suppression $\propto \exp(-Q')$. Therefore $\frac{n_N}{n_{Fe}}$ is doubly suppressed, and hence efficient break up of α particles requires a higher temperature.

The dissociation of ^{56}Fe leads to an instability in the core. Intuitively, it happens at the expense of energetic photons whose disappearence lower the pressure. Quantitatively, the break up results in a decrease in the adiabatic index γ. Recall that the core will be unstable for $\gamma < \frac{4}{3}$. We can estimate how the photodisintegration affects γ.

For adiabatic changes:

$d\varepsilon = -P\,dv$ $\quad$ (ε: specific energy, v: specific volume)

From definition:

$$\gamma = \left(\frac{\partial \ln P}{\partial \ln \rho}\right)_{ad} = \frac{-v}{P}\left(\frac{\partial P}{\partial v}\right)_{ad}$$

Now:

$$d\varepsilon = \left(\frac{\partial \varepsilon}{\partial v}\right)_P dv + \left(\frac{\partial \varepsilon}{\partial P}\right)_v dP \Rightarrow \left(\frac{\partial \varepsilon}{\partial v}\right)_P + \left(\frac{\partial \varepsilon}{\partial P}\right)\left(\frac{\partial P}{\partial v}\right)_{ad} = -P$$

It is therefore seen that:

$$\gamma = \frac{Pv + v\left(\frac{\partial \varepsilon}{\partial v}\right)_P}{P\left(\frac{\partial \varepsilon}{\partial P}\right)_v}$$

Assuming that ε is a function of Pv only (as in most cases), we obtain:

$$\gamma = 1 + \frac{1}{\frac{\partial \varepsilon}{\partial (Pv)}} \simeq 1 + \frac{1}{\frac{\Delta \varepsilon}{\Delta (Pv)}}$$

During the break up of ^{56}Fe, which is an adiabatic process, we have:

$\Delta E \simeq 2 \times 10^{18}$ erg g^{-1}

On the other hand:

$P V = \frac{k_B T}{\tilde{m}}$ ($\tilde{m}$: mean mass per particle)

Thus:

$(P V)_{before} = \frac{k_B T}{56 m_U}$ (^{56}Fe before break up)

$(P V)_{after} = \frac{k_B T}{\frac{56}{17} m_U}$ (13α and $4n$ after break up)

This results in:

$\Delta(P V) = (P V)_{after} - (P V)_{before} = \frac{2}{7} \frac{k_B T}{m_U} = 2.4 \times 10^{17}$ erg g^{-1} ($T \simeq 10^{10}$ K)

It is therefore seen that:

$$\boxed{\gamma \simeq 1 + \frac{\Delta(P V)}{\Delta E} \simeq 1.1 < \frac{4}{3}}$$

This indicates an instability due to photodisintegration. We saw the same thing happening during the protostar formation when Hydrogen molecules break up and H, He atoms are ionized (see page 143). The physical reason is the same in both cases. Energy gained by the release of gravitational potential energy during the collapse goes into break up instead of increasing temperature and, subsequently, pressure.

Neutronization also leads to an instability. Electron capture by protons means lowering the degeneracy pressure of the electron gas. One can do a similar analysis by estimating the adiabatic index γ during neutronization. The result is:

$$\gamma = \gamma_0 (1 - \delta)$$

Where γ_0 is the adiabatic index without neutronization (i.e. $\gamma_0 = \frac{4}{3}$ in the extreme relativistic limit). It turns out that $\delta > 0$, and hence $\gamma < \frac{4}{3}$ again.

Next we will discuss neutrino mean free path in the collapsing core. As we will see, the core becomes opaque to neutrinos at densities above $\gtrsim 10^{11}\ g\,cm^{-3}$. Neutrinos get trapped at such high densities, and further electron capture will be blocked because of the Pauli principle.